POSITIVITY OF BOUNDARY INTERSECTIONS OF COMPLEX CURVES

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Abstract: Let an almost complex structure J of class $\mathcal{C}^1$ in a neighborhood of the origin in $\mathbb{R}^{2n}$ be given, $n \geq 2$. Let furthermore $0 \in W$ be a germ of a J-totally real submanifold of real dimension n and of class $\mathcal{C}^2$. After an appropriate $\mathcal{C}^2$-smooth coordinate change we can assume wlg that for an appropriate neighborhood B of zero we have that $(B, W \cap B) = (\mathbb{R}^{2n}, \mathbb{R}^n)$ and $J|_{\mathbb{R}^n} = J_{st}$. Here J_{st} stands for the standard complex structure of $\mathbb{C}^n$. Such coordinate change we shall call a **redressing map**. Denote by $\Delta^+ := \{\zeta \in \Delta : \Im\zeta \geq 0\}$ the upper half-disk and by $\beta_0 := (-1, 1)$ its edge. Let a J-holomorphic map $u : (\Delta^+, \beta_0, 0) \to (\mathbb{R}^{2n}, \mathbb{R}^n, 0)$ be given, assume it is continuous up to β_0. We shall call such u a J-complex half-disk attached to W. It is not difficult to prove that there exist a $\mu \in \mathbb{N}$ such that

$$u(\zeta) = \mathrm{v}_0\zeta^\mu + O(|\zeta|^{\mu+\alpha}) \quad \text{with} \quad \mathrm{v}_0 \neq 0.$$

We shall call v_0 the **tangent vector** to u at zero and the number μ we shall call the **order of vanishing** of u at zero. This μ doesn't depend on the redressing map. Making a reflection with respect to $W = \mathbb{R}^n$, i.e., setting

$$\tilde{u}(\zeta) = \begin{cases} u(\zeta) \text{ if } \Im\zeta \geq 0 \\ \overline{u(\bar{\zeta})} \text{ if } \Im\zeta < 0, \end{cases}$$

we can extend u to Δ as a $\mathcal{C}^\alpha$-regular map. Now let $u_1, u_2 : (\Delta^+, \beta_0, 0) \to (\mathbb{R}^4, \mathbb{R}^2, 0)$ be two J-complex disks attached to W which intersect at zero. We define the boundary intersection index of u_1 and u_2 at zero as the intersection number at zero of their extensions by reflection, and denote this index as $\mathrm{ind}_0^{\mathrm{b}}(u_1, u_2)$. Our goal is present the following

Theorem 1.*Let J be a $\mathcal{C}^1$-regular almost-complex structure on $\mathbb{R}^{2n}$ such that $J|_{\mathbb{R}^n} = J_{st}$ and let $u_i : (\Delta^+, \beta_0, 0) \to (\mathbb{R}^4, \mathbb{R}^2, 0)$ be two J-complex half-disks such that one is not a reparameterization of another. Then their boundary intersection index is correctly defined and satisfies*

$$\mathrm{ind}_0^{\mathrm{b}}(u_1, u_2) \geq \mu_1 \cdot \mu_2,$$

where μ_i *is the order of vanishing of* u_i *at zero,* $i = 1, 2$.

Remark. To our best knowledge this result is new even for integrable J unless W is supposed to be real analytic.